Optimal Power Allocation for Hybrid Overlay/Underlay Spectrum Sharing in Multiband Cognitive Radio Networks

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***Abstract*—In this paper, we consider power allocation in multiband cognitive radio (CR) networks, where multiple secondary users (SUs) transmit via a common relay and compete for the transmit power of the relay. We employ a hybrid overlay/underlay spectrum sharing scheme, allowing the SU to adapt its way of accessing the licensed spectrum to the status of the primary user (PU). If the PU is detected to be idle at the selected channel, the SU works in an overlay mode; else, it works in spectrum underlay. In addition, an auction-based power-allocation scheme is proposed to solve power competition of multiple SUs. For the SU working in spectrum overlay, the relay allocates the power in proportion to its payment without additional constraints; for the SU in spectrum underlay, its own transmit power and that of the relay are upper bounded for the quality of service (QoS) of the PU. Then, the convergence of the proposed auction algorithm and the outage probability of secondary transmissions is theoretically analyzed. Finally, the performance of the proposed scheme is verified by the simulation results.**

***Index Terms*—Auction game, cognitive radio (CR), hybrid spectrum sharing, outage probability, power allocation.**

I. Introduction

WITH the rapid deployment of wireless services over the last decade, the radio spectrum is becoming a valuable and scarce resource. How to support growing applications with limited spectrum resources emerges as a critical issue for future wireless communications. On the other side, the report from the Federal Communications Commission reveals that most of the licensed spectrum is severely underutilized [1]. As a promising technique, cognitive radio (CR) [2], [3] is proposed to deal with the dilemma between spectrum scarcity and spectrum underutilization. CR allows unlicensed users [referred to as secondary users (SUs)] to access licensed bands under the condition that the induced interference to the licensed users [referred to as primary users (PUs)] does not reach an unacceptable level.

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In general, an SU has three spectrum sharing approaches: 1) *Opportunistic spectrum access* (also known as spectrum overlay) [4], under which a SU accesses a band only when it is not being used by the PU; 2) *Spectrum sharing* (also known as spectrum underlay) [5], where the SU coexists with the PU and transmits with power constraints to guarantee the quality of service (QoS) of the PU; and 3) *Sensing-based spectrum sharing* [6], with which the SU first senses the status of the PU (idle/active) and then selects an appropriate spectrum sharing mode based on the sensing result. If the PU is detected to be active, the SU selects the spectrum underlay mode and transmits with lower power. Otherwise, the SU works at spectrum overlay and transmits with its maximum power budget for a higher data rate. In such hybrid overlay/underlay scheme, the SU can transmit in both the idle and busy bands. It improves the throughput of the secondary network while maintaining a harmless interference to the PU. In this paper, we study distributed power allocation for SUs in hybrid CR networks, where multiple SUs simultaneously transmit under different spectrum sharing modes with the help of a common relay and compete for the relay's transmit power.

As a promising spectrum sharing scheme, sensing-based spectrum sharing not only improves the throughput of the secondary network but also guarantees the QoS of the primary network. However, the studies in the literature on such hybrid spectrum sharing CR networks are still relatively sparse. An adaptive modulation framework was proposed in [7], which supports multicarrier-based signals to generate hybrid overlay/underlay waveforms for a soft-decision CR. In [8], Oh and Choi proposed a hybrid CR system, where the switch from an overlay mode to an underlay mode is probabilistically controlled to maximize the departure rate of the SU. In [9], Bao *et al.* considered the maximum transmission capacity of the SUs under the primary and secondary outage constraints for a hybrid CR network. The energy-efficient transmission for hybrid spectrum sharing CR networks was investigated in [10], and an optimization model was established to maximize the energy-efficiency capacity. For a relay-assisted hybrid CR system, power control for the SUs and for the relay is a critical issue. This paper considers a distributed power bidding and allocation algorithm for maximizing the data rate of the SUs under the primary QoS constraints. To the best of our knowledge, the power auction for hybrid spectrum sharing is reported for the first time in this paper.

Research on power control for CR networks has been conducted most recently [11]–[14]. For instance, Ghasemi and Sousa [11] proposed a power-allocation scheme in a fading environment. It maximizes the ergodic capacity of the SU, given the interference temperature constraint at the primary receiver. In [12], an opportunistic power control strategy was proposed for SUs. It protects the PU's transmission and realizes spectrum sharing between the PU and the SUs. In [13] and [14], the authors studied how to design the sensing time and allocate the power for maximizing the ergodic throughput of a CR system. Most previous work formulated the power allocation as convex or linear programming problems. In practice, their solutions often require global information and coordination among all users, which is very costly and sometimes even infeasible in distributed settings.

Game theory is a simple and powerful tool for distributed resource allocation in interactive multiuser systems [15]–[24]. In [15], the SNR auction and the power auction schemes were proposed to coordinate the relay power allocation among users based on amplify-and-forward relaying protocol. Wang *et al.* [16] proposed a two-level buyer/seller game for cooperative communications, in which the interests of source nodes and relay nodes are jointly considered. To address both system efficient and user fairness issues of CR networks, Yang *et al.* [17] proposed a distributed power control strategy by using a cooperative Nash bargaining game model. In [18], a joint power-and-rate control strategy was presented for SUs based on a cooperative game theoretic framework. In [19], Etkin *et al.* considered distributed self-enforcing strategies for coexistence in unlicensed bands and applied noncooperative game theory to find fair and efficient solutions. Using cooperative game theory, Attar *et al.* [20] developed an optimum strategy for resource allocation in a secondary spectrum-access scenario. For multimedia streaming over CR networks, three auction-based schemes were proposed in [21] to realize distributed spectrum allocation. Auction and pricing mechanisms were also investigated in [22]–[24] for efficient spectrum allocation.

In a hybrid spectrum sharing scheme, although the SU is allowed to work in two spectrum sharing modes, different power bidding and allocation strategies should be employed for different modes. If the SU works in spectrum overlay, the relay allocates the power to the SU in proportion to its payment without additional constraints. Meanwhile, the SU is allowed to transmit with its power budget for a higher throughput; for the SU in spectrum underlay, in addition to its own transmit power constraint, the power assigned by the relay is constrained for the QoS performance of the PU. When using the existing game theoretic methods (e.g., [15]–[17]), the SUs cannot be differentiated in accordance with their spectrum sharing mode. Instead, they would be treated equally. Thus, the existing methods are perfect for stand-alone (nonhybrid) spectrum sharing cases but are not well suited for the proposed hybrid system.

Cooperative communication [25], [26], which is a new spatial diversity technique, has great potential to be used in CR networks to combat channel fading and to improve the throughput. In [27] and [28], cooperative diversity in detection of the PU was applied, and it was shown that and the detection time can be reduced greatly through the cooperation between the SUs. In [29] and [30], the secondary transmitter (ST) was allowed to act as a relay for the primary transmissions, which showed that the throughput of the secondary network can be improved in certain network topologies. A joint cooperation diversity scheme and best relay selection were proposed in [31] for multirelay CR networks, which not only improves the performance of secondary transmissions but guarantees the QoS of primary transmissions as well. However, relay-assisted secondary transmissions in a joint overlay-and-underlay spectrum sharing CR network has never been touched in the literature.

In this paper, we address the power-allocation problem in hybrid overlay/underlay CR networks, where multiple SUs transmit via a common relay and compete for the transmit power of the relay. First, we propose a relay power-allocation scheme on the basis of an auction game, in which the relay organizes an auction for selling its transmit power, whereas the SU acts as the player and bids for a maximum utility. Second, we mathematically prove the convergence [i.e., the convergence to a unique Nash equilibrium (NE)] of the proposed auction algorithm. Finally, we derive the closed form of the outage probability for both spectrum sharing modes.

Our main contributions are summarized as follows.

1) A hybrid overlay/underlay spectrum sharing scheme is employed for multiband CR networks, where the SU adapts its way of accessing the licensed spectrum to the status of the PU. If the PU is idle at the channel selected by the SU, the SU works in an overlay mode; otherwise, the SU works in spectrum underlay, under which its transmit power and relay power are constrained to avoid causing harmful interference to the PU.
2) A distributed power bidding and allocation algorithm for relay-assisted secondary transmissions is explicitly developed for multiuser CR networks.
3) The outcome of the proposed auction game is investigated, and the existence of a unique NE is theoretically proven.
4) In the proposed hybrid system, the SU would have different outage probability in different spectrum sharing modes, which is difficult to be directly derived from the existing results. In this paper, we provide theoretical analysis of outage probability for a hybrid spectrum sharing system, which supports the SUs working in two modes coexisted.

The remainder of this paper is organized as follows. Section II presents the network model and the basic assumptions. Section III describes a power auction mechanism for multiuser CR networks and analyzes its convergence performance. In Section IV, the outage probability of secondary transmissions is theoretically analyzed. Numerical results are presented and discussed in Section V. Finally, Section VI concludes this paper.

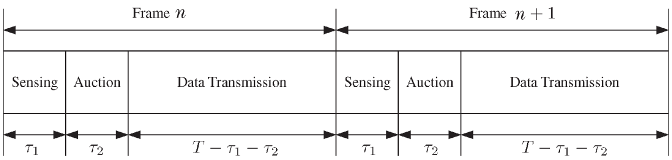


Fig. 1. Structure of the CR frame.

II. Network Modeling and Notations

Consider a CR system consisting of a primary network divided into M nonoverlapping narrowband channels, and a secondary network composed of $N(N < M)$ secondary links. In the primary network, there is a primary transmitter (PT) and a primary destination (PD). In the secondary network, there are N STs, N secondary destinations (SDs), and a secondary relay (SR) assisting ST transmissions by decode-and-forward (DF) protocol. Assume that each channel of the primary network can be accessed by only one ST, and the channel occupancy by the STs is maintained by the SR.

The structure of our CR frame is shown in Fig. 1. It consists of a sensing slot, an auction slot, and a data transmission slot. The sensing slot is for spectrum sensing and channel allocation. In the auction slot, the STs join the power auction organized by the SR, and bid for the transmit power of the SR. Thereafter, the STs use the remaining frame duration $T-\tau_1-\tau_2$ for data transmission. At the beginning of the sensing slot, the ST, which intends to send data to its SD, randomly selects a channel, detects the status of the PU at this channel, and then submits the detection decision to the SR through an error-free control channel. To avoid data collision at the SR, the STs can send their decisions in a time-division multiplexing manner.

The SR uses one bit flagging the occupancy of a channel by the ST, which is initialized to "0." When receiving the detection result from an ST, the SR checks the flag of this channel. If the flag is "1," i.e., the channel has already been allocated to another ST, the SR informs the ST of choosing other channels. Else, the SR allocates this channel to the ST, updates the channel flag to "1," and switches to the next ST. In this paper, we assume an accurate detection. In addition, we assume that the change of PU's status at each channel only occurs at the beginning of a CR frame, and this status is assumed to be static within the frame. Although these assumptions simplify our analysis, they do not limit the main conclusions of this paper to be used as a baseline analysis for more complicated scenarios.

If the PU is idle at the channel assigned to the ST, the ST works in a spectrum overlay fashion shown in Fig. 2(a). Otherwise, the ST works together with the PU in spectrum underlay, as shown in Fig. 2(b). Here, the solid lines indicate the intended communications, whereas the dotted lines represent the interference.

We assume a slow-fading channel, such that the channel is stable within each frame. Let G_A^B denote the channel gains from A to B, which is also the channel gains from B to A. The amplitude $|G_A^B|^2$ is exponentially distributed, with rate parameter $\lambda_A^B = (d_A^B)^\alpha$, where d_A^B denotes the distance between A and B, and α is the path-loss exponent. Assume that all the channel gains are available to the ST. For slow-fading channel, i.e., the channel coherence time is large enough, the channel gains can be accurately estimated within a sufficiently long period of observation.

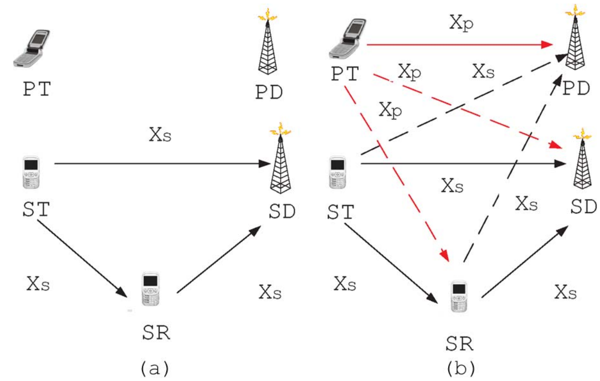


Fig. 2. Network model.

a) Spectrum overlay mode:

In the first phase of the data transmission slot: ST_i transmits its data to the SR and its destination SD_i. Their received signals are given by

$$Y_{\mathrm{ST}_i}^{\mathrm{SR}} = \sqrt{P_i} G_{\mathrm{ST}_i}^{\mathrm{SR}} X_{\mathrm{ST}_i} + n_{\mathrm{SR}} \tag{1}$$
$$Y_{\mathrm{ST}_i}^{\mathrm{SD}_i} = \sqrt{P_i} G_{\mathrm{ST}_i}^{\mathrm{SD}_i} X_{\mathrm{ST}_i} + n_{\mathrm{SD}_i} \tag{2}$$

where Y_A^B represents the signal received at B from A, X_{ST_I} is information symbols transmitted by ST_i with $E[|X_{\mathrm{ST}_i}|^2] = 1$, P_i is the transmit power budget of ST_i, and $n_{\{\cdot\}}$ is the additive white Gaussian noise with variance σ^2.

In spectrum overlay (with superscript "00"), the SNR of X_{ST_i} at SD_i in the first phase is

$$\Gamma_i^{00}(1) = \frac{P_i \left| G_{\mathrm{ST}_i}^{\mathrm{SD}_i} \right|^2}{\sigma^2}. \tag{3}$$

In the second phase: The SR decodes X_{ST_I} from $Y_{\mathrm{ST}_i}^{\mathrm{SR}}$ and forwards it to SD_i. Then, the signal received at SD_i is

$$Y_{\mathrm{SR}}^{\mathrm{SD}_i} = \sqrt{P_{r_i}} G_{\mathrm{SR}}^{\mathrm{SD}_i} X_{\mathrm{ST}_i} + n'_{\mathrm{SD}_i} \tag{4}$$

where P_{ri} is the SR's transmit power for ST ST_i.

The SNR of $X_{\mathrm{ST}i}$ at SD_i is then

$$\Gamma_i^{00}(2) = \frac{P_{r_i} \left| G_{\mathrm{SR}}^{\mathrm{SD}_i} \right|^2}{\sigma^2}. \tag{5}$$

Therefore, in spectrum overlay, the achievable rate from ST_i to SD_i via the SR is

$$R_i^{00} = \frac{1}{2} W \log_2 \left(1 + \Gamma_i^{00}(1) + \Gamma_i^{00}(2)\right) \tag{6}$$

where W is the signal bandwidth. The factor 1/2 comes from the fact that two phases are required to fulfill one transmission in this relay-assisted communication. In addition, τ_1 and τ_2 are assumed to be negligible for simplicity.

b) Spectrum underlay mode:

i: *In the first phase:* In this case, the PT and ST_i transmit simultaneously, and their transmissions affect each other. Thus, the signal received at the SR and SD_i are

$$Y_{\mathrm{ST}_i}^{\mathrm{SR}} = \sqrt{P'_i} G_{\mathrm{ST}_i}^{\mathrm{SR}} X_{\mathrm{ST}_i} + \sqrt{P_u} G_{\mathrm{PT}}^{\mathrm{SR}} X_{\mathrm{PT}} + n_{\mathrm{SR}} \tag{7}$$
$$Y_{\mathrm{ST}_i}^{\mathrm{SD}_i} = \sqrt{P'_i} G_{\mathrm{ST}_i}^{\mathrm{SD}_i} X_{\mathrm{ST}_i} + \sqrt{P_u} G_{\mathrm{PT}}^{\mathrm{SD}_i} X_{\mathrm{PT}} + n_{\mathrm{SD}_i} \tag{8}$$

where P_i' is ST's maximum transmit power allowed in spectrum underlay, P_u is PT's transmit power, and $X_{\rm PT}$ is the signal transmitted by the PT in this phase with $E[|X_{\rm PT}|^2]=1$.

Thus, the signal-to-interference ratio (SINR) of $X_{{\rm ST}_i}$ at ${\rm SD}_i$ under spectrum underlay (with superscript "10") is

$$\gamma_i^{10}(1)=\frac{P_i'\left|G_{{\rm ST}_i}^{{\rm SD}_i}\right|^2}{P_u\left|G_{\rm PT}^{{\rm SD}_i}\right|^2+\sigma^2}. \tag{9}$$

For the primary link, the signal received by the PD is

$$Y_{\rm PT}^{\rm PD}(1)=\sqrt{P_u}G_{\rm PT}^{\rm PD}X_{\rm PT}+\sqrt{P_i'}G_{{\rm ST}_i}^{\rm PD}X_{{\rm ST}_i}+n_{\rm PD}. \tag{10}$$

The SINR of $X_{\rm PT}$ at the PD is given by

$$\gamma_p^{10}(1)=\frac{P_u\left|G_{\rm PT}^{\rm PD}\right|^2}{P_i'\left|G_{{\rm ST}_i}^{\rm PD}\right|^2+\sigma^2}. \tag{11}$$

The rate achieved by the PU in the first phase is

$$R_p^{10}(1)=\frac{1}{2}W\log_2\left(1+\gamma_p^{10}(1)\right). \tag{12}$$

ii: *In the second phase:* The SR decodes $X_{{\rm ST}_i}$ from $Y_{{\rm ST}_i}^{\rm SR}$ and forwards it to ${\rm SD}_i$. Then, the signal received by SD_i is

$$Y_{\rm SR}^{{\rm SD}_i}=\sqrt{P_{r_i}}G_{\rm SR}^{{\rm SD}_i}X_{{\rm ST}_i}+\sqrt{P_u}G_{\rm PT}^{{\rm SD}_i}X_{\rm PT}+n'_{{\rm SD}_i}.$$

Therefore, the SINR of $X_{{\rm ST}_i}$ at ${\rm SD}_i$ is

$$\gamma_i^{10}(2)=\frac{P_{r_i}\left|G_{\rm SR}^{{\rm SD}_i}\right|^2}{P_u\left|G_{\rm PT}^{{\rm SD}_i}\right|^2+\sigma^2}. \tag{13}$$

For the primary link, the signal received by the PD is

$$Y_{\rm PT}^{\rm PD}(2)=\sqrt{P_u}G_{\rm PT}^{\rm PD}X_{\rm PT}+\sqrt{P_{r_i}}G_{\rm SR}^{\rm PD}X_{{\rm ST}_i}+n'_{\rm PD}. \tag{14}$$

Thus, the SINR of $X_{\rm PT}$ at the PD is

$$\gamma_p^{10}(2)=\frac{P_u\left|G_{\rm PT}^{\rm PD}\right|^2}{P_{r_i}\left|G_{\rm SR}^{\rm PD}\right|^2+\sigma^2}. \tag{15}$$

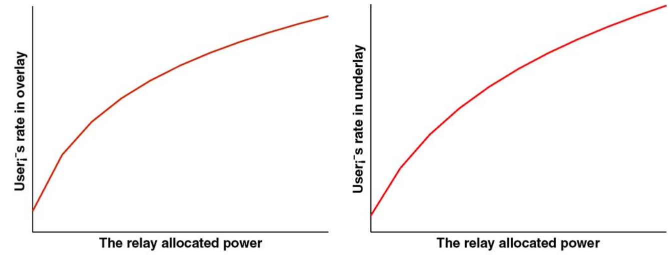
The rate achieved by the PU in the second phase is

$$R_p^{10}(2)=\frac{1}{2}W\log_2\left(1+\gamma_p^{10}(2)\right). \tag{16}$$

Thus, in spectrum underlay, the achievable rate of ${\rm ST}_i$ is

$$R_i^{10}=\frac{1}{2}W\log_2\left(1+\gamma_i^{10}(1)+\gamma_i^{10}(2)\right). \tag{17}$$

For any ${\rm ST}_i$, the relationships between its achievable rate R_i^{00} or R_i^{10} and the transmit power P_{r_i} obtained from the SR are shown in Fig. 3. It is observed that the achievable rate R_i^{00} and R_i^{10} are both nondecreasing functions of the relay transmit power P_{r_i}. The more transmission power received from the relay, the higher the achievable rate would be. This leads to power competition among multiple STs at the relay.



(a) (b)

Fig. 3. Achievable rate versus relay transmission power. (a) R_i^{00} versus relay transmission power. (b) R_i^{10} versus relay transmission power.

III. Power Auction Mechanism

Here, we first present a power-allocation scheme based on an auction game, where the SR allocates the transmission power for the STs in accordance with their payments. Then, we theoretically analyze the convergence performance of the proposed auction game.

A. Power Auction Game

If the ST works in spectrum overlay, it is allowed to transmit with the maximum power, i.e., its transmit power budget P_i, for a higher data rate. If the ST works in spectrum underlay, it adapts the transmit power to P_i' to provide QoS guarantee for the PU, i.e.,

$$R_p^{10}(1)\geq R_{\rm req}\Rightarrow P_i' =\min\left\{\frac{1}{\left|G_{{\rm ST}_i}^{\rm PD}\right|^2}\left(\frac{P_u\left|G_{\rm PT}^{\rm PD}\right|^2}{M}-\sigma^2\right),P_i\right\} \tag{18}$$

where $R_{\rm req}$ is a minimum rate required for the PU, and $M\triangleq 2^{2R_{req}/W}-1$.

In spectrum underlay, not only the transmit power of the ST but its maximum power $\widehat{P_{r_i}}$ received from the SR should be restricted for the performance of the primary transmission, i.e.,

$$R_p^{10}(2)\geq R_{\rm req}\Longrightarrow\widehat{P_{r_i}}=\frac{1}{\left|G_{\rm SR}^{\rm PD}\right|^2}\left(\frac{P_u\left|G_{\rm PT}^{\rm PD}\right|^2}{M}-\sigma^2\right). \tag{19}$$

During the auction game, ${\rm ST}_i$ (player i) iteratively submits its bid $f_i(t)$ to the SR; then, the SR updates the power assigned to ${\rm ST}_i$ in proportion to its payment $f_i(t)P_{r_i}(t)$, i.e.,

$$P_{r_i}(t+1)=\min\left\{\widehat{P_{r_i}},\frac{f_i(t)P_{r_i}(t)}{\sum_{j\in\mathcal{N}}\left(f_j(t)P_{r_j}(t)\right)}P_r\right\} \tag{20}$$

where t is the iteration index, P_r is the total power of the SR, and

$$\widehat{P_{r_i}}=\begin{cases}P_r, & \text{Overlay mode}\\ \frac{1}{|G_{\rm SR}^{\rm PD}|^2}\left(\frac{P_u|G_{\rm PT}^{\rm PD}|^2}{M}-\sigma^2\right), & \text{Underlay mode.}\end{cases} \tag{21}$$

To depict a ST's satisfaction with the power received from the relay, we define a utility function for ${\rm ST}_i$ as

$$U_i(t)=gR_i(t)-f_i(t)P_{r_i}(t) \tag{22}$$

where g is a positive constant providing conversion of units, $R_i(t)=R_i^{00}(t)$ in the overlay mode, and $R_i(t)=R_i^{10}(t)$ in the underlay mode.

Definition 1: The optimal power profile $\mathbf{P}_\mathbf{r}^* = (P_{r_1}^*, \ldots, P_{r_N}^*)$ is the desirable outcome of a power auction game, with which any ST ST_i achieves the maximum utility, i.e.,

$$U_i\left(P_{r_i}^*; \mathbf{P}_{\mathbf{r}_{\underline{\mathbf{i}}}}^*\right) \geq U_i(P_{r_i}; \mathbf{P}_{\mathbf{r}_{\underline{\mathbf{i}}}}) \quad \forall i \in \mathcal{N}$$

where $\mathbf{P}_{\mathbf{r}_{\underline{\mathbf{i}}}} \triangleq (P_{r_1}, \ldots, P_{r_{i-1}}, P_{r_{i+1}}, \ldots, P_{r_N})$, which we call the supplementary power profile of P_{r_i}. When the optimal power profile $\mathbf{P}_\mathbf{r}^*$ occurs, the game reaches an NE.

According to [32], an auction game $\langle \mathcal{N}, P_{r_i}, U_i \rangle$ has an NE if, for all $i \in \mathcal{N}$, 1) the assigned power set $\mathbf{P}_{\mathbf{r}_\mathbf{i}}$ of player i is a nonempty compact convex subset of a Euclidian space, and 2) the payoff function U_i is continuous and quasi-concave on P_{r_i}.

Theorem 1: The proposed auction game in (22), with the optimal power profile in Definition 1, has an NE.

Proof: At any tth iteration, the proof is suitable; therefore, we omit t in the following proof.

It is straightforward to show (1) for the feasible power set. To show (2), it is clear that the utility function $U_i(P_{r_i})$ is continuous; therefore, we just prove that the utility function is quasi-concave.

Given f_i, we differentiate the utility function in (22) with respect to P_{r_i}, and yield

$$\begin{aligned}\frac{\partial U_i(P_{r_i})}{\partial P_{r_i}} &= g\frac{\partial R_i}{\partial P_{r_i}} - f_i\\ &= \begin{cases} W\frac{A_0}{2ln2A_1} - f_i, & \text{Overlay mode}\\ W\frac{B_0}{2ln2B_1} - f_i, & \text{Underlay mode.}\end{cases}\end{aligned} \tag{23}$$

Furthermore, we have

$$\frac{\partial^2 U_i(P_{r_i})}{\partial^2 P_{r_i}} = \begin{cases} -W\frac{(A_0)^2}{2ln2(A_1)^2} \leq 0, & \text{Overlay mode}\\ -W\frac{(B_0)^2}{2ln2(B_1)^2} \leq 0, & \text{Underlay mode}\end{cases} \tag{24}$$

where

$$\begin{aligned} A_0 &= \frac{\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{\sigma^2}\\ A_1 &= 1 + \frac{P_i\left|G_{\mathrm{ST}_i}^{\mathrm{SD}_i}\right|^2}{\sigma^2} + \frac{P_{r_i}\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{\sigma^2}\\ B_0 &= \frac{\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2 + \sigma^2}\\ B_1 &= 1 + \frac{P_i'\left|G_{\mathrm{ST}_i}^{\mathrm{SD}_i}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2 + \sigma^2} + \frac{P_{r_i}\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2 + \sigma^2}. \end{aligned}$$

Thus, the utility function $U_i(P_{r_i})$ is a concave function of P_{r_i}, and the existence of the NE is established. ■

B. Distributed Iterative Algorithm

Before the game starts, each ST initializes the required original power $P_{r_i}(0)$ and its bid $f_i(0)$ and then submits them to the relay. To get a higher rate, the ST would request relay's power as much as possible. Therefore, $P_{r_i}(0)$ would be initialized to $\widehat{P_{r_i}}$ by all the STs. To satisfy $\partial U_i(P_{r_i})/\partial P_{r_i} = 0$, the ST would set the original bid $f_i(0)$ as

$$f_i(0) = \begin{cases} W\frac{A_0}{2\ln 2A_2}, & \text{Overlay mode}\\ W\frac{B_0}{2\ln 2B_2}, & \text{Underlay mode}\end{cases} \tag{25}$$

where

$$\begin{aligned} A_0 &= \frac{\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{\sigma^2}\\ B_0 &= \frac{\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2 + \sigma^2}\\ A_2 &= 1 + \frac{P_i\left|G_{\mathrm{ST}_i}^{\mathrm{SD}_i}\right|^2}{\sigma^2} + \frac{\widehat{P_{r_i}}\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{\sigma^2}\\ B_2 &= 1 + \frac{P_i'\left|G_{\mathrm{ST}_i}^{\mathrm{SD}_i}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2 + \sigma^2} + \frac{\widehat{P_{r_i}}\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2 + \sigma^2}. \end{aligned}$$

In each iteration, the ST updates its bid $f_i(t)$ in such a way that its utility $U_i(t)$ satisfies

$$\begin{aligned}\frac{\partial U_i(t+1)}{\partial P_{r_i}(t+1)} = 0 &\Rightarrow \frac{\partial(gR_i(t+1) - f_{i+1}(t)P_{r_i}(t+1))}{\partial P_{r_i}(t+1)} = 0\\ &\Rightarrow \frac{\partial gR_i(t+1)}{\partial P_{r_i}(t+1)} - f_i(t+1) = 0.\end{aligned} \tag{26}$$

When rearranging (26), we can obtain (27), shown at the bottom of the page, where

$$G_i = \begin{cases} 0, & \text{Overlay mode}\\ 1, & \text{Underlay mode,}\end{cases} \qquad \widetilde{P}_i = \begin{cases} P_i, & \text{Overlay mode}\\ P_i', & \text{Underlay mode.}\end{cases}$$

$$\begin{aligned} f_i(t+1) &= \begin{cases} \frac{W}{2ln2}\dfrac{\frac{\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{\sigma^2}}{1+\frac{P_i\left|G_{\mathrm{ST}_i}^{\mathrm{SD}_i}\right|^2}{\sigma^2}+\frac{P_{r_i}(t+1)\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{\sigma^2}}, & \text{Overlay mode}\\ \frac{W}{2ln2}\dfrac{\frac{\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2+\sigma^2}}{1+\frac{P_i'\left|G_{\mathrm{ST}_i}^{\mathrm{SD}_i}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2+\sigma^2}+\frac{P_{r_i}(t+1)\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2+\sigma^2}}, & \text{Underlay mode}\end{cases}\\ &= \frac{W}{2ln2}\frac{\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2}{G_iP_u\left|G_{\mathrm{PT}}^{\mathrm{SD}_i}\right|^2 + \sigma^2 + \widetilde{P}_i\left|G_{\mathrm{ST}_i}^{\mathrm{SD}_i}\right|^2 + P_{r_i}(t+1)\left|G_{\mathrm{SR}}^{\mathrm{SD}_i}\right|^2} \end{aligned} \tag{27}$$

It is observed that the iteration in (27) is a distributed implementation, in which the update of the bid only requires local information. In addition, it is found that $\partial f_i(t+1)/\partial P_{r_i}(t+1) \leq 0$; thus, $f_i(t+1)$ is a nonincreasing function of $P_{r_i}(t+1)$. The more power is assigned, the lower the bid will be. This depicts the desirability of the ST. Therefore, the upper bound $\widehat{f}_i$ of $f_i(t)$ is achieved by setting $P_{r_i} = 0$, and the lower bound $\breve{f}_i$ of $f_i(t)$ corresponds to the maximum power assignment $\widehat{P_{r_i}}$. It is worth noticing that when the ST initializes its required power $P_{r_i}(0)$ to the upper bound $\widehat{P_{r_i}}$, the yielded bid $f_i(0)$ actually equals to the minimum bid $\breve{f}_i$, i.e., $f_i(0) = \breve{f}_i$.

Lemma 1: If $\mathbf{f} \geq \mathbf{f}'$, then $F_i(\mathbf{f}) \geq F_i(\mathbf{f}')$.

Proof: When the auction game reaches the NE, the ST obtains the desired power and no longer needs to update its bid, i.e., the bid satisfies $f_i(t+1) \equiv f_i(t)$. Combining (20) and (27), we have

$$\begin{aligned} f_i(t+1) &= \begin{cases} \frac{1}{P_r - P_{r_i}(t)} \sum_{j \neq i} f_j(t) P_{r_j}(t), & \text{If } P_{r_i}(t+1) \neq \widehat{P_{r_i}}; \\ f_i(0), & \text{Otherwise} \end{cases} \\ &= F_i(f_1(t), \ldots, f_{i-1}(t), f_{i+1}(t), \ldots, f_N(t)) \\ &\triangleq F_i(\mathbf{f}(t)). \end{aligned} \tag{28}$$

We adopt the convention that the vector inequality $\mathbf{f} > \mathbf{f}'$ is a strict inequality in all components. When performing partial derivative of the iterative function in (28) with respect to $f_j(t)$, $j \in \{1, \ldots i-1, i+1, \ldots, N\}$, we have

$$\frac{\partial f_i(t+1)}{\partial f_j(t)} = \begin{cases} \frac{P_{r_j}(t)}{P_r - P_{r_i}(t)}, & \text{If } P_{r_i}(t+1) \neq \widehat{P_{r_i}} \\ 0, & \text{Otherwise.} \end{cases} \tag{29}$$

Since $P_{r_j}(t) > 0$ and $P_{r_i}(t) < P_r$, we have $\partial f_i(t+1)/\partial f_j(t) \geq 0$; therefore, the monotonicity holds for the iterative function in (28). ■

Theorem 2: The bid iterative function in (28), with the original bid $\mathbf{f}(0) = \breve{\mathbf{f}}$ where $\breve{\mathbf{f}} \triangleq (\breve{f}_1, \ldots, \breve{f}_N)$, converges to a unique fixed point.

Proof: Since $\mathbf{f}(0) = \breve{\mathbf{f}}$, we have

$$\mathbf{f}(1) \geq \mathbf{f}(0). \tag{30}$$

Applying Lemma 1 to the given inequality for $k-1$ times, we have

$$\mathbf{f}(k) \geq \mathbf{f}(k-1). \tag{31}$$

Therefore, the sequence of the bid vector is nondecreasing. Further, the bid sequence is upper bounded by $\widehat{f}_i$. The sequence would stop increasing at some k only when

$$\mathbf{f}(k) = \boldsymbol{F}(\mathbf{f}(k-1)) = \mathbf{f}(k-1). \tag{32}$$

■

A complete power bidding and allocation algorithm is shown in Algorithm 1. Considering that the SR cannot always successfully decode the ST's signal, the ST is required to transmit before the auction starts. If the signal of the ST is not decoded correctly by the SR, i.e., the SR fails to help the SR at the moment, the ST would not join the current auction and can try again in the next frame. For the ST whose data are successfully recovered at the SR, it is allowed to participate in the current auction. During the auction game, the bid update and power-allocation processes are iterated in an alternating way, until the auction game converges to the optimum.

Algorithm 1 Power bidding and allocation algorithm

Step 1. Request for cooperation

$\mathbf{ST}_i$: sends a request to the relay for cooperation.

SR: responds the cooperation request of ST_i.

$\mathbf{ST}_i$: transmits the data in the first phase.

SR: If the received signals cannot be successfully decoded, it inform ST_i of the failure; else, it allows ST_ ST_i to participate in the auction and goes to Step 2.

Step 2. Initialization

$\mathbf{ST}_i$: initializes the required power $P_{r_i}(0)$ to $\widehat{P_{r_i}}$ by (21), calculates the original bid $f_i(0)$ by (25), and then submits these values to the relay.

Step 3. Power Allocation

SR: updates the allocated power $P_{r_i}(t+1)$ for all the STs by (20), then informs the STs.

Step 4. Bid Update

$\mathbf{ST}_i$: updates its bid $f_i(t+1)$ according to (27) and sends it back to the relay.

Step 5. Convergence

Repeat Step 3 and Step 4, until the value of $f_i(t)$ no longer changes with additional iterations.

IV. Outage Probability

Without loss of generality, we consider that an outage of secondary transmissions occurs when SD_i fails to recover the information from ST_i. In this paper, we assume that the SR assists the ST transmissions by DF protocol. If the SR fails to decode the STs signal, i.e., the outage of the transmissions from the ST to the SR occurs, the SR cannot help the ST at the moment. Thus, the ST would not participate in the current auction. Only the ST whose data are successfully decoded at the SR is allowed to join the auction. Here, we investigate the impact of the proposed relay power-allocation scheme on the performance of the outage probability of secondary transmissions (secondary outage probability). Thus, we only consider the outage of the transmissions from the SR to the SD and assume that the transmission from the ST to the SR has not suffered the outage.

For overlay and underlay mode, the mutual information between the ST and the SD are defined, respectively, as

$$I_i^{00} = \frac{1}{2}\log_2\left(1 + \Gamma_i^{00}(1) + \Gamma_i^{00}(2)\right) \tag{33}$$

$$I_i^{10} = \frac{1}{2}\log_2\left(1 + \gamma_i^{10}(1) + \gamma_i^{10}(2)\right). \tag{34}$$

Meanwhile, the mutual information between the ST and the SR are defined, respectively, as

$$I_{r_i}^{00} = \frac{1}{2}\log_2\left(1 + \frac{P_i\left|G_{\mathrm{ST}_i}^{\mathrm{SR}}\right|^2}{\sigma^2}\right) \tag{35}$$

$$I_{r_i}^{10} = \frac{1}{2}\log_2\left(1 + \frac{P_i'\left|G_{\mathrm{ST}_i}^{\mathrm{SR}}\right|^2}{P_u\left|G_{\mathrm{PT}}^{\mathrm{SR}}\right|^2 + \sigma^2}\right). \tag{36}$$

Definition 2: The outage probability $P(R)$ is the probability of the mutual information I_i falling below a certain rate R, i.e., $P(R) = P(I_i < R)$.

Theorem 3: The secondary outage probability in the overlay mode is $(1 + (\lambda_v/\lambda_u\rho - \lambda_v)e^{-\lambda_u\Delta} - (\lambda_u\rho/\lambda_u\rho - \lambda_v)e^{-\lambda_v\Delta/\rho})e^{-\lambda_t\Delta}$, where $t \triangleq |G^{\mathrm{SR}}_{\mathrm{ST}_i}|^2$, $u \triangleq |G^{\mathrm{SD}_i}_{\mathrm{ST}_i}|^2$, $v \triangleq |G^{\mathrm{SD}_I}_{\mathrm{SR}}|^2$, $\rho \triangleq P_{r_i}/P_i$, $\chi \triangleq P_I/\sigma^2$, and $\Delta \triangleq (2^{2R} - 1)/\chi$.

Proof: Assume that the channels experience independent fading, and t, um and v are independently exponentially distributed with parameters λ_t, λ_u, and λ_v, respectively. Then, we have

$$\begin{aligned} P\left(I_i^{00}<R\right) &= P\{u+\rho v<\Delta\} \\ &= \int_0^{\frac{\Delta}{\rho}} \left[1-e^{-\lambda_u(\Delta-\rho v)}\right]\lambda_v e^{-\lambda_v v}dv \\ &= 1+\frac{\lambda_v}{\lambda_u\rho-\lambda_v}e^{-\lambda_u\Delta} - \frac{\lambda_u\rho}{\lambda_u\rho-\lambda_v}e^{-\lambda_v\frac{\Delta}{\rho}}. \end{aligned} \quad (37)$$

Meanwhile, we have

$$P\left(I_{r_i}^{00} > R\right) = P\{t>\Delta\} = e^{-\lambda_t\Delta}. \quad (38)$$

Therefore, we have

$$\begin{aligned} P_i^{00} &= P\left(I_i^{00}<R, I_{r_i}^{00}>R\right) \\ &= P\left(I_i^{00}<R\right)P\left(I_{r_i}^{00}>R\right) \\ &= \left(1+\frac{\lambda_v}{\lambda_u\rho-\lambda_v}e^{-\lambda_u\Delta}-\frac{\lambda_u\rho}{\lambda_u\rho-\lambda_v}e^{-\lambda_v\frac{\Delta}{\rho}}\right)e^{-\lambda_t\Delta}. \end{aligned} \quad (39)$$

■

Theorem 4: The secondary outage probability in the underlay mode is $\lambda_u/(\lambda_u+\lambda_x/\Delta'\beta)\lambda_v/(\lambda_v+\lambda_x\rho'/\Delta'\beta)\lambda_y/(\lambda_y + \lambda_t\Delta'\beta)e^{\lambda_x/\beta}e^{-\lambda_t\Delta'}$, where $t \triangleq |G^{\mathrm{SR}}_{\mathrm{ST}_i}|^2$, $u \triangleq |G^{\mathrm{SD}_i}_{\mathrm{ST}_i}|^2$, $v \triangleq |G^{\mathrm{SD}_i}_{\mathrm{SR}}|^2$, $x \triangleq |G^{\mathrm{SD}_i}_{PU}|^2$, $y \triangleq |G^{\mathrm{SR}}_{PU}|^2$, $\beta \triangleq P_u/\sigma^2$, $\rho' \triangleq P_{r_i}/P'_i$, $\chi' \triangleq P'_i/\sigma^2$, and $\Delta' \triangleq (2^{2R} - 1)/\chi'$.

Proof: Assume that the channels experience independent fading and that t, u, v, x, and y are independently exponentially distributed with parameters λ_t, λ_u, λ_v, λ_x, and λ_y. Thus, we have

$$\begin{aligned} &P\left(I_i^{10} < R | I_{r_i}^{10} > R\right) \\ &\quad = P\{u+\rho' v < \Delta'\beta x + \Delta'\} \\ &\quad = P\left\{\frac{u+\rho' v-\Delta'}{\Delta'\beta} < x\right\} \\ &\quad = \int_0^\infty\int_0^\infty e^{-\lambda_x\frac{u+\rho' v-\Delta'}{\Delta'\beta}}\lambda_u e^{-\lambda_u u}\lambda_v e^{-\lambda_v v}dudv \\ &\quad = \frac{\lambda_u}{\lambda_u+\frac{\lambda_x}{\Delta'\beta}}\frac{\lambda_v}{\lambda_v+\frac{\lambda_x\rho'}{\Delta'\beta}}e^{\frac{\lambda_x}{\beta}} \end{aligned} \quad (40)$$

$$\begin{aligned} &P\left(I_{r_i}^{10} > R\right) \\ &\quad = P\{t > \Delta'\beta y + \Delta'\} \\ &\quad = \int_0^\infty e^{-\lambda_t(\Delta'\beta y+\Delta')}\lambda_y e^{-\lambda_y y}dy \\ &\quad = \frac{\lambda_y}{\lambda_y+\lambda_t\Delta'\beta}e^{-\lambda_t\Delta'}. \end{aligned} \quad (41)$$

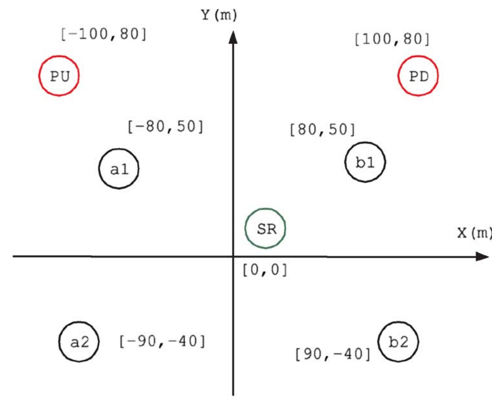


Fig. 4. Two-user simulation network.

Thus, we have

$$\begin{aligned} P_i^{10} &= P\left(I_i^{10} < R | I_{r_i}^{10} > R\right) P\left(I_{r_i}^{10} > R\right) \\ &= \frac{\lambda_u}{\lambda_u+\frac{\lambda_x}{\Delta'\beta}}\frac{\lambda_v}{\lambda_v+\frac{\lambda_x\rho'}{\Delta'\beta}}\frac{\lambda_y}{\lambda_y+\lambda_t\Delta'\beta}e^{\frac{\lambda_x}{\beta}}e^{-\lambda_t\Delta'}. \end{aligned} \quad (42)$$

■

V. Simulation Results

Here, we present simulation results to demonstrate the performance of the proposed power-allocation algorithm. We first consider a scenario shown in Fig. 4, where two secondary links $(a_1 \to b_1, a_2 \to b_2)$ attempt to access the spectrum of the PU. The channel gains are $(0.097/d^\alpha)^{1/2}$, where d is the distance between two nodes, and the path-loss exponent is $\alpha = 4$. Without a special specification, the transmit power budget of each ST is set to 0.01 W, the transmit power of the PU is 0.01 W, the total power of the relay is $P_r = 0.2$ W, and the noise variance is $\sigma^2 = 10^{-13}$.

Fig. 5 displays the relationship between the SU's achievable utility and the PU's predefined rate R_{req} under four spectrum sharing scenarios. If both SUs work in the overlay mode, as shown in Fig. 5(a), their utilities remain invariant, regardless of the value of R_{req}. The reason is that when the PU is idle, the SUs do not need to consider the QoS requirement of the PU. Fig. 5(b) and (c) shows the cases that one SU works in overlay, and the other works in underlay. It is observed that, for the user who is in underlay, its utility decreases with the increase in R_{req}. The larger R_{req} is, the smaller P'_i and $\widehat{P_{r_i}}$ are; therefore, there will be less transmit power and relay power for this user. On the other hand, the smaller $\widehat{P_{r_i}}$ for the user in underlay, there will be more relay power remaining for the user in overlay. Thus, the utility of the user in overlay increases with R_{req}. In Fig. 5(d), two SUs are both in the underlay mode. It is clear that their utilities decrease with R_{req}.

Fig. 6 plots the impact of the transmit power budget P_i of the ST on the SU's achievable utility under four spectrum sharing scenarios. The PU's predefined rate R_{req} is set to 0.4. As shown in Fig. 6(a), when both users are in spectrum overlay, they can transmit with their power budget P_i, regardless of the constraint of the PU. In Fig. 6(b), user 1 works in overlay; therefore, its utility increases with the increase in its power budget. While for user 2, which is in underlay, we find that its utility increase with P_i at the beginning but remains unchanged when P_i reaches 0.5.

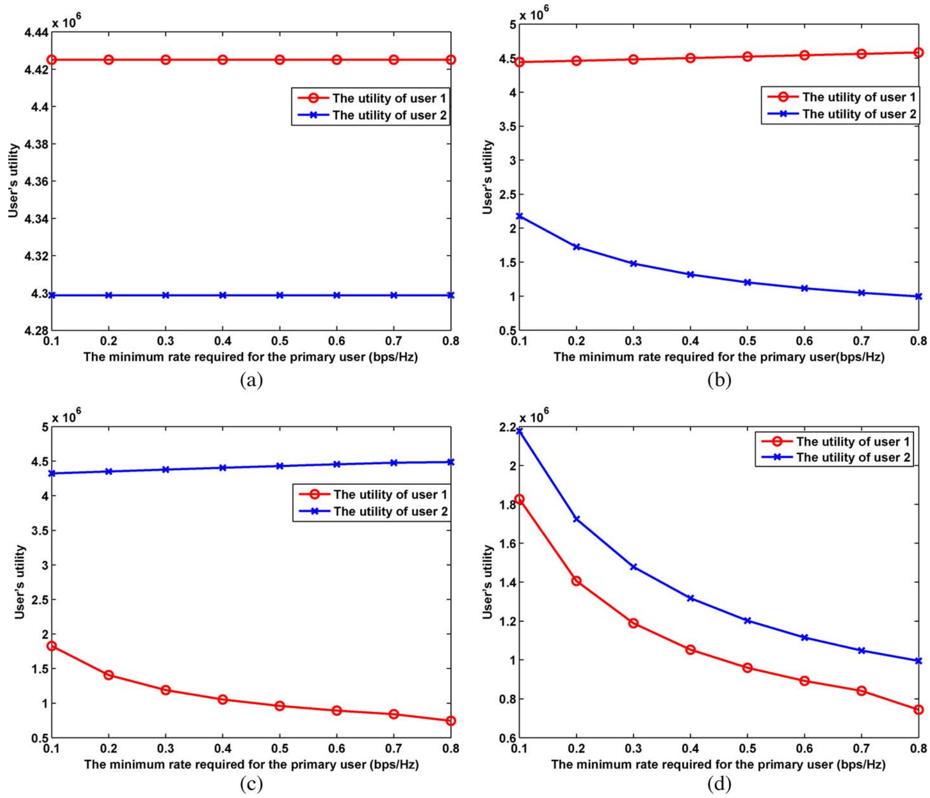


Fig. 5. User utility in four spectrum sharing scenarios. (a) Both users in overlay. (b) User 1 in overlay and user 2 in underlay. (c) User 1 in underlay and user 2 in overlay. (d) Both users in underlay.

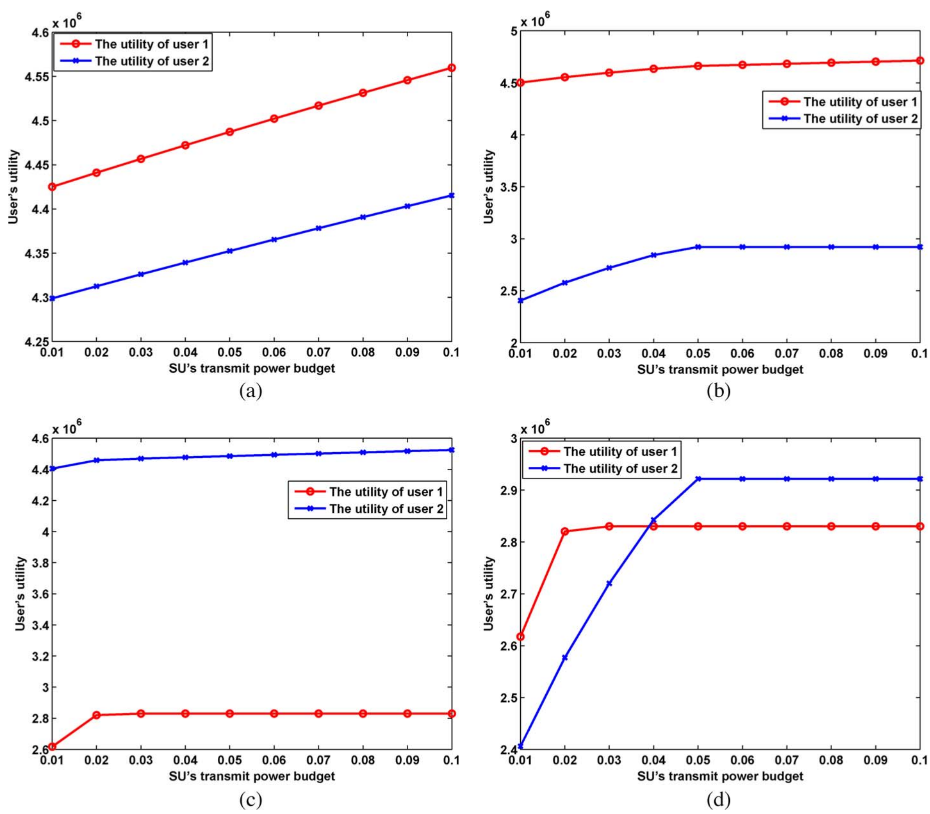


Fig. 6. Four possible combinations. (a) Both users in overlay. (b) User 1 in overlay anad user 2 in underlay. (c) User 1 in underlay and user 2 in overlay. (d) Both users in underlay.

At that point, the available transmit power of the user reaches maximum, i.e., P_i equals $\widehat{P}_i$. Fig. 6(c) shows the situation that user 1 is in underlay, whereas user 2 is in overlay, and the result is similar to Fig. 6(b). In Fig. 6(d), these two users are both in underlay, and their utilities increase with P_i and keep static when P_i reaches some certain value.

TABLE I
SIMULATION COMPARISON FOR OVERLAY MODE

Rate R	Analytical P_{out}	Simulation P_{out}
0.5	1.3359e-004	1.3401e-004
1.0	1.1287e-003	1.1304e-003
1.5	5.4314e-003	5.4154e-003
2.0	1.9689e-002	1.9653e-002
2.5	5.4452e-002	5.4452e-002

TABLE II
SIMULATION COMPARISON FOR UNDERLAY MODE

Rate R	Analytical P_{out}	Simulation P_{out}
0.5	8.2104e-003	8.2192e-003
1.0	4.3858e-002	4.3793e-002
1.5	1.2196e-001	1.2159e-001
2.0	2.4296e-001	2.4171e-001
2.5	3.6851e-001	3.6490e-001

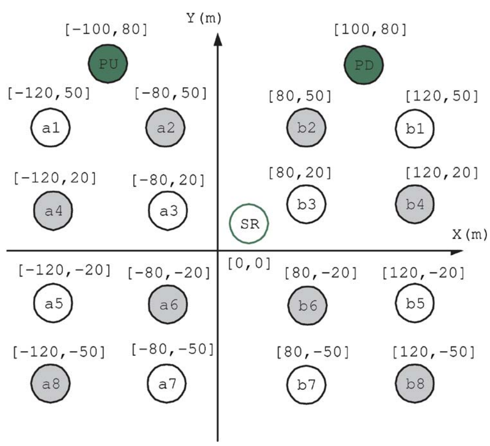


Fig. 7. Eight-user simulation network.

We now compare the analytical expression of outage probability derived in Theorems 3 and 4 with the simulation results. Without loss of generality, we only need to simulate the outage event for one user with a given relay power P_{r_i} as the outage probability calculation is applicable to all the users. We conduct simulation with 1×10^7 realizations, and the simulation results for user a_1 can be found in Tables I and II for the overlay and underlay modes, respectively. We see a perfect match between the simulation results and the analytical results, and we thus conclude that our analysis is accurate in calculating the outage probability.

Next, we increase the number of the SUs in a CR network to eight. As shown in Fig. 7, we assume that a_1, a_2, a_3, and a_4 are in overlay mode, whereas a_5, a_6, a_7, and a_8 are in underlay mode.

Fig. 8 shows the convergence performance of the proposed power-allocation algorithm. It is seen that, for users $a_5 \sim a_8$, who are working in underlay mode, their bids remain the same all the time. Constrained by the QoS requirement of the PU, the assigned power of these four users are always equal to the maximum power $\widehat{P_{r_i}}$ that they can get from the SR, which results in the unchanged power allocation and the unchanged bids. For users $a_1 \sim a_4$ in overlay mode, their bids converge at a very fast speed, which means that all eight users converge to the optimum bids within five iterations.

Fig. 9 compares the achievable rate via cooperation transmission (CT) with that in direct transmission (DT). We find that for all eight users, the CT under the proposed power allocation

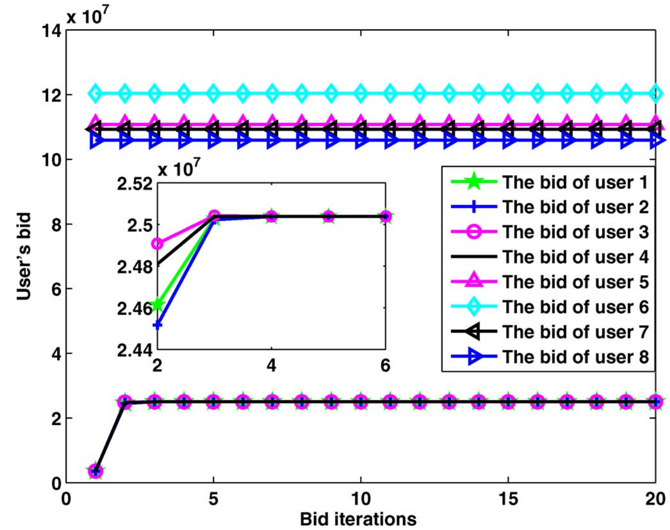


Fig. 8. Convergence of user bid.

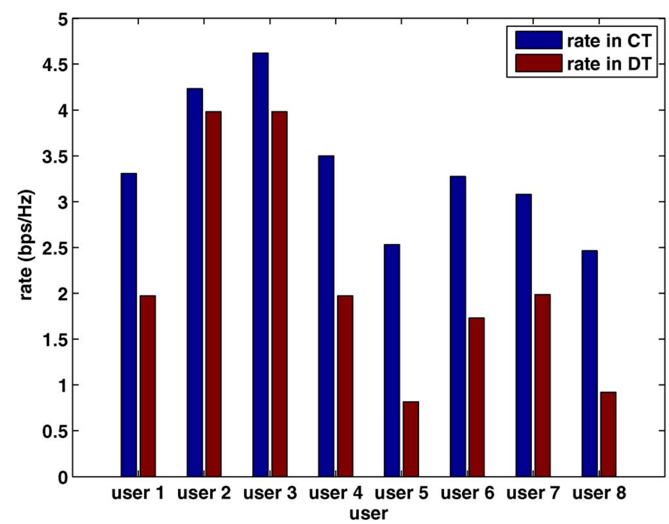
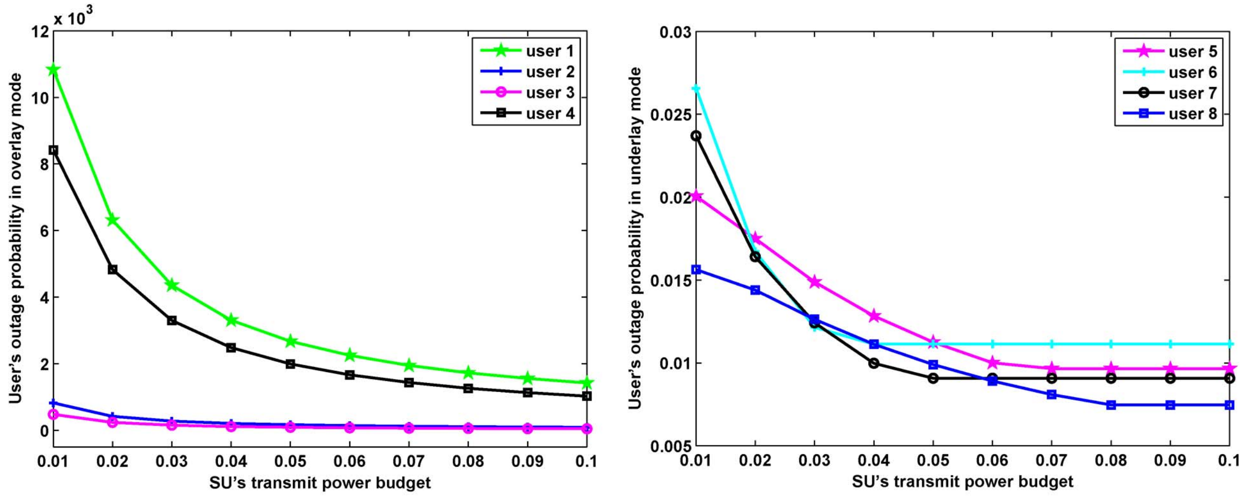


Fig. 9. User rate in CT and DT.

outperforms the DT. For the ST whose signal cannot be decoded correctly by the SR, it can choose DT as an alternative.

Finally, we demonstrate the performance of the secondary outage probability.

We consider the relationship of the secondary outage probability and the ST's transmit power budget P_i. The threshold R is set to 1, the minimum rate R_{req} of the PU is 0.4, and the ST's transmit power budget P_i varies within the range (0.01 W, 0.1 W). Fig. 10(a) shows the variations of the outage probability of user $a_1 - a_4$ who are working in overlay mode. It is observed, for a given rate R, that the outage probability of these four users decrease with the increase in their transmit power. Thus, the larger the transmit power, the higher data rate the user can achieve, and the smaller the outage probability will be. Fig. 10(b) shows the variations of the outage probability of $a_5 - a_8$ who are in underlay mode. We find that their outage probability at first decreases with P_i and remains static when P_i reaches a certain value. This is due to the fact that, at that point, the available transmit power of the user reaches maximum, i.e., P_i equals $\widehat{P_i}$. Even if we continually increase the user transmit power budget, its available transmit power is fixed, thus leading to an invariant outage probability.



(a) (b)

Fig. 10. Secondary outage probability versus user transmit power budget. (a) Outage probability of $a_1 - a_4$. (b) Outage probability of $a_5 - a_8$.

VI. Conclusion

Sensing-based spectrum sharing combines the benefits of both spectrum overlay and spectrum underlay to improve the throughput of the SU, without generating harmful interference to the PU. In this paper, we have tackled the power-allocation problem for relay-assisted secondary transmissions in a hybrid overlay/underlay spectrum sharing CR network, where the SUs join the power auction organized by the relay and bid for maximizing the utility. The auction algorithm and its convergence performance are investigated. Future work can be extended to the cases with imperfect spectrum sensing and will find a way to mitigate the interference between the PU and the SUs.

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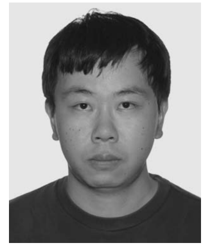

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